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A study on L-fuzzy open compact topology

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Abstract. In this article, the notion of L-fuzzy locally compact space and L-fuzzy evaluation maps are introduced and further L-fuzzy covering map between two L-fuzzy spaces are studied. Also, the concept of L-fuzzy exponential law and some related theories are proved. Finally, the L-fuzzy open compact topology is introduced and some results of L-fuzzy open compact topology are explained.

MSC : 54A40, 54E55.

Key words :L-Fuzzy space, L-Fuzzy hausdorff space, L-Fuzzy compact spaces and L-Fuzzy open compact topology.

1 Introduction

Zadeh added the essential standards of fuzzy sets in his classical paper [1]. Fuzzy sets have lots of applications in fields of engineering, social technological know-how, economics, clinical science and many others,. In mathematics, topology furnished the most natural framework for the ideas of fuzzy units to flourish. Chang [2] added and advanced the concept of fuzzy L-fuzzy topological spaces. The essential organization of fuzzy topological areas was brought by using Abdul Razak Salleh and Mohammad faucet in [5] and [7]. prompted through [5] and [7], fuzzy essential organization in fuzzy topological areas became prolonged to numerous fuzzy structure spaces in [8] and [4]. The concept of exponential law can be used in case of KU algebra [6].

In general topology, the concept of compact-open topology is crucial for defining function spaces [9]. The goal of this paper is to introduce the notion of fuzzy compact-open topology and to present various theories and results related to it. In this paper, a new concept of L-Fuzzy locally compact space and L-Fuzzy evaluation maps are introduced and further L-Fuzzy covering map between two fuzzy L-Fuzzy spaces are studied. Also, the concept of fuzzy L-Fuzzy exponential law and some theories are proved. Finally L-Fuzzy open compact topology is introduced and some results of L-Fuzzy open compact topology are proved. In this study, we have established the concepts of L-fuzzy local compactness and L-fuzzy product topology which are very pivotal.

2 Preliminaries

In terms of notation, we use letters like A, B, C, U, V, W, and so on to represent fuzzy sets. The set of all fuzzy sets on a nonempty set X is denoted as I^X. The constant fuzzy sets with the values 0 and 1 on X are denoted by 0_X and 1_X, respectively. The fuzzy closure, fuzzy interior, and fuzzy complement of $A \in I^X$ will be denoted by $\tilde{A}$, A^o, andA', respectively. For our future use, we'll need the definitions and findings listed below.

Definition 2.1. [1] A function from a non-empty set X to a unit interval $I = [0, 1]$ is called a fuzzy set λ. I^X denotes the fuzzy set family as a whole.

Definition 2.2. [3] Let X be a set and τ be a family of fuzzy subsets of X. Then τ is called fuzzy topology on X if satisfies the following conditions:

(i) $0_X, 1_X \in \tau$

(ii) If $\lambda, \mu \in \tau$ then $\lambda \wedge \mu \in \tau$

(iii) If $\lambda_i \in \tau$ for all I then $\vee\lambda_i \in \tau$.

Definition 2.3. For any fuzzy set $A \in F(X)$ and any $\lambda \in [0, 1]$, the λ-cut and strong λ-cut of A are respectively defined as follows: $A_\lambda = \{x \in X : A(x) \geq \lambda\}, A_{\langle\lambda\rangle} = \{x \in X : A(x) \geq \lambda\}$, where $A(x) = \mu A(x)$ since $A(x)$ is more convenient than $\mu A(x)$.

Definition 2.4. Let I^τ be set of all monotonic decreasing maps $\lambda : \mathbb{R} \to L$ (where L is completely distributive lattice) satisfying:

(i) $\lambda(t) = 1$ for $t < 0$,

(ii) $\lambda(t) = 0$ for $t > 1$.

For $\lambda, \mu \in I^\Gamma$, we define that $\lambda \equiv \mu$ iff $\lambda(t-) = \mu(t-)$ and $\lambda(t+) = \mu(t+)$ for all $t \in \mathbb{R}$, where $\lambda(t-) = \inf_{s<t}\lambda(s)$ and $\lambda(t+) = \sup_{s>t}\lambda(s)$. Then $\equiv$ is an equivalence relation on $I^\Gamma, [\lambda]$ denotes the equivalence class of $\lambda \in I^\Gamma$ and the quotient set $I^\Gamma/\equiv$ is called the L- fuzzy unit interval which in symbols is written $I(L)$.

We define an L-fuzzy topology τ on $I(L)$ by taking as a subbase $\{L_{t,R_t:t\in\mathbb{R}}\}$, where we define $L_t([\lambda]) = (\lambda(t-))'$ and $R_t([\lambda]) = (\lambda(t+))'$. The topology τ is called the standard topology on $I(L)$, and the base of τ is $\{L_s \wedge R_t : s, t \in \mathbb{R}\}$.

Definition 2.5. Let $f, g : (X, \tau) \to (Y, \sigma)$ be L-fuzzy continuous maps. We say that f is L-fuzzy homotopic to g if there exists an L-fuzzy continuous map $F : (X, \tau) \times (I(L), \tau) \to (Y, \sigma)$ such that $F(a_\alpha, [\lambda_0]) = f(a_\alpha)$ and $F(a_\alpha, [\lambda_1]) = g(a_\alpha)$ for every L-fuzzy point $a_\alpha \in (X, \tau)$ where $i = 0, 1$.

$$\lambda_i(t) = \begin{cases} 1, & t < i \\ 0, & t > i \end{cases}$$

The map F is called an L-fuzzy homotopy between f and g, and written $F : f \cong_L g$.

Definition 2.6. Let (X,τ) and (Y,σ) be any two L-fuzzy space. Let $p:(X,\tau)\to(Y,\sigma)$ is called a L-fuzzy covering space if and only if

(*i*) p is L-fuzzy onto.

(*ii*) For every $a_\alpha \in X$ there exists a neighborhood $a_\alpha \in U$ such that $p^{-1}(u) = \cup S_i$ such that each s_i is L-fuzzy homeomorphic to U.

Each s_i is called a sheet. Each u for which $p^{-1}(u) = \cup s_i$ is said to be L-fuzzy covered. $p^{-1}(a_\alpha)$ is called a L-fuzzy fiber.

Definition 2.7. Let $p:(X,\tau)\to(Y,\sigma)$ be a L-fuzzy covering map and let $f:(\tilde{X},\tilde{\sigma})\to(Y,\sigma)$ be a L-fuzzy continuous function. Then a map $\tilde{f}:(\tilde{X},\tilde{\sigma})\to(X,\tau)$ is said to be L-fuzzy lift on the map f if $p\circ\tilde{f} = f$.

3 L-Fuzzy locally compact space

Definition 3.1. Let (X,τ) be a L-fuzzy topological space. The family of fuzzy sets A of X is said to be a cover of a fuzzy set B in X if and only if $B = \vee A$. If each member of the cover is a member of τ, then this cover is said to be an open cover of B. The L-fuzzy topological space (X,τ) is called L-fuzzy compact if every fuzzy open cover of I^X has a finite L-fuzzy open subcover.

Example 3.2. Let (X,τ) be the unit interval I with τ topology and $\tau = \{0_X, 1_X, \lambda_X, \mu_X\}$. Consider $\lambda_X(x) = \{x \in I : 0 \le x \le 0.7\}$ and $\lambda_X(y) = \{y \in I : 0.7 \le y \le 1\}$. So clearly $\{\lambda_X(x), \lambda_X(y)\}$ be open cover of X. Now define $\forall x \in I, x \ne 0, x \ne 1$, let $\mu_X(x) = 1$ and $\forall y \in \left[0, \frac{x}{2}\right] \cup \left[\frac{1+x}{2}, 1\right]$, $\mu_X(y) = 0$. So $\{\mu_X(x), \mu_X(y)\}$ be the subcover of open cover of X. Thus (X,τ) be L-fuzzy compact space.

Definition 3.3. (X,τ) is called L-fuzzy Hausdorff space if any two distinct fuzzy points, i.e., x_t and y_t, there exists L-fuzzy open sets U and V such that $x_t \in U$, $y_t \in V$ and $U \wedge V = 0_X$.

Example 3.4. Let $X = \{x, y\}$ and $\lambda \in I^X$ be defined as $\lambda(x) = 0.5$, $\lambda(y) = 0.6$. Then $\tau = \{0_X, 1_X, \lambda\}$. Here $\tau \cup 0_X$ forms a L-fuzzy on X. Clearly (X,τ) is a L-fuzzy Hausdorff space.

Theorem 3.5. *A L-fuzzy compact subspace of a L-fuzzy Hausdorff space is L-fuzzy closed.*

Proof. Let (X,τ) be a L-fuzzy Hausdorff topological space and $Y \subset X$, Y is L-fuzzy compact. We have to show that Y is L-fuzzy closed, i.e., $(X-Y)$ is L-fuzzy open. Let $x_t \in (X-Y)$ be arbitrary. For each $y_t \in Y$, $x_t \ne y_t$, since X is L-fuzzy Hausdorff there exists L-fuzzy open sets U and V such that $x_t \in U, y_t \in V$ and $U \cap V = 0_X$. The collections $\{V_y : y \in Y\}$ is a L-fuzzy open cover for Y. Since Y is L-fuzzy compact there exists a finite sub cover $\{V_{y_1}, V_{y_2} \cdots V_{y_n}\}$ such that $Y \subseteq V_{y_1} \cup V_{y_2} \cdots V_{y_n} = V$. Let $U = U_{y_1} \cap U_{y_2} \cap \cdots U_{y_n}$ then U and V are L-fuzzy open in X. Clearly $U \cup V = 0_X$. And $Y \subset V$ implies $(X-V) \subset (X-Y)$. Now $x_0 \in U \subset (X-V) \subset (X-Y)$. So $(X-Y)$ is L-fuzzy closed. $\square$

Theorem 3.6. *The following criteria are equivalent in a L-fuzzy Hausdorff space:*

(a) *X is L-fuzzy locally compact.*

(b) *For every L-fuzzy point $x_t \in X$, there exists a L-fuzzy open set U in X such that $x_t \in U$ and $\bar{U}$ is L-fuzzy compact.*

Proof. Assume that X is L-fuzzy locally compact. So for every L-fuzzy point $x_t \in X$, there exists a L-fuzzy open set U in X which is L-fuzzy compact. U is L-fuzzy closed as X is L-fuzzy Hausdorff. Thus $U = \bar{U}$. Hence $x_t \in U$ and $\bar{U}$ is L-fuzzy compact.

Assume that if for every L-fuzzy point $x_t \in X$, there exists a L-fuzzy open set U in X such that $x_t \in U$ and $\bar{U}$ is L-fuzzy compact then obviously X is L-fuzzy locally compact. □

Theorem 3.7. *The L-fuzzy Hausdorff space X is L-fuzzy locally compact at a L-fuzzy point x_t in X if and only if for each L-fuzzy open set U containing x_t, there exists an L-fuzzy open set V such that $x_t \in V$, $\bar{V}$ is L-fuzzy compact and $\bar{V} \subseteq U$.*

Proof. Assume that X is L-fuzzy locally compact at an L-fuzzy point x_t. So there exists a L-fuzzy open set U such that $x_t \in U$ and U is L-fuzzy compact. Again it is given that X is L-fuzzy Hausdorff. So U is L-fuzzy closed; thus $U = \bar{U}$. Let us consider a fuzzy point $y_r \in (1_X - U)$. Since X is L-fuzzy Hausdorff, there exist open sets A and B such that $x_t \in A$ and $y_r \in B$ and $A \cap B = 0_X$. Let $V = A \cap B$. Hence $V \subseteq U$ implies $\bar{V} \subseteq \bar{U} = U$. Since $\bar{V}$ is L-fuzzy closed and U is L-fuzzy, it follows that $\bar{V}$ is L-fuzzy compact. Thus $x_t \in \bar{V} \subseteq U$ and $\bar{U}$ is L-fuzzy compact. The converse follows from previous theorem. □

4 L-Fuzzy compact open topology

Let (X, τ) and (Y, σ) be any two L-fuzzy generated by τ and σ.

Let

$$Y^X = \{f : (X, \tau) \to (Y, \sigma)| \ f \text{ is } L- \text{ fuzzy } \text{ continuous function.}\}$$

We give this class Y^X a topology called the L-fuzzy compact open topology as follows:

Let

$$\kappa : \{K : I \to X : K \text{ is } L - \text{fuzzy compact } \text{in} X\}$$

$$\eta = \{U : I \to Y \text{ such that } U \text{ is } L - \text{fuzzy open } \text{ in } Y\}.$$

For any $K \in \kappa$ and $U \in \eta$, let

$$W(K, U) = \left\{\omega \in Y^X : \omega(K) \subseteq U\right\}$$

The collection $\{W(K, U) : K \in \kappa, U \in \eta\}$ can be as a L-fuzzy sub base to generate a L-fuzzy topology on the class Y^X, called the L-fuzzy compact-open topology. The class Y^X with this topology is called a L-fuzzy compact-open topological space. Unless otherwise stated, Y^X will always have the L-fuzzy compact-open topology.

We now consider the L-fuzzy product topological space $Y^X \times C$ and define a L-fuzzy continuous map from $Y^X \times X$ into Y.

Definition 4.1. The L-fuzzy evaluation map is the mapping $f' : Z^W \times W \to Z$ defined by $f'(f, \lambda_t) = f(\lambda_t)$ for each fuzzy point $\lambda_t \in W$ and $f \in Z^W$ is called L-fuzzy evaluation map.

Theorem 4.2. *Let (W, η) be a L-fuzzy locally compact Hausdorff space. Then the L-fuzzy evaluation map $f' : Z^W \times W \to Z$ is L-fuzzy continuous.*

Proof. Suppose $(f, \lambda_t) \in Z^W \times W$ where $f \in Y^W$ and $\lambda_t \in W$. Let V be a L-fuzzy open set in Z that has $f(x) = f'(f, \lambda_t)$. By Theorem 3.3, there exists a L-fuzzy open set U in W such that $\lambda_t \in U$, $\bar{U}$ is L-fuzzy compact and $f(U) \subseteq V$, because W is fL-fuzzy locally compact and f is L-fuzzy continuous.

Consider the $N_{\bar{U} \cdot V} \times U$ L-fuzzy open set in $Z^W \times W$. Clearly $(f, \lambda_t) \in N_{\bar{U} \cdot V} \times U$. Assume $(g, \lambda_r) \in N_{\bar{U} \cdot V} \times U$ is random; hence $f'(g, \lambda_r) = g(x_r) \in V$ where $\lambda_r \in U, g(\lambda_r) \in V$. As a result, $f'(g, \lambda_r \times U) \subseteq V$, indicating that f' is L-fuzzy continuous. □

We now consider the induced map of a given function $f : Z' \times W \to Z$.

Definition 4.3. Let W, Z, Z' denote L-fuzzy space and $f : Z' \times W \to Z$ denote any function. For fuzzy point $\lambda_t \in W$ and $z'_r \in Z'$ the induced map $\hat{f} : W \to Z^{Z'}$ is defined by $(f(\lambda_t))(z'_r) = f(z'_r, \lambda_t)$ for fuzzy points . In the case of a function $\hat{f} : W \to Z^{Z'}$ a comparable function f can be found by the same rule.

$\hat{f}$'s continuity can be described in terms of f's continuity, and vice versa. For this, we require the following outcome.

Theorem 4.4. *Consider two L-fuzzy, W and Z, with Z L-fuzzy compact. Let O be a L-fuzzy open set in the L-fuzzy product space and λ_t be any fuzzy point in W. $\lambda_t \times Z$ is contained in $W \times Z$. Then there exists some L-fuzzy neighborhood S of λ_t in W such that $\{\lambda_t\} \times Z \subseteq S \times Z \subseteq O$.*

Proof. $\lambda_t \times Z$ is clearly L-fuzzy homeomorphic to Z, and hence $\{\lambda_t\} \times Z$ is L-L-fuzzy compact. We cover $\{\lambda_t\} \times Z$ with the basis elements $U \times V$ (L-fuzzy topology of $W \times Z$) residing in O. Since $\lambda_t \times Z$ is L-fuzzy compact, $\{U \times V\}$ has a finite subcover,i.e.,

$$U_1 \times V_1 \times \cdots \times U_n \times V_n.$$

Without loss of generality we'll suppose that $\lambda_t \in U_i$ for each $i = 1, 2 \cdots n$. Suppose

$$S = \bigcap_{i=1}^{n} U_i$$

. Clearly S is L-fuzzy open and $\lambda_t \in S$. We have to show that

$$S \times Z \subseteq \bigcup_{i=1}^{n} (U_i \times V_i).$$

Any fuzzy point in $S \times Z$ can be represented as (λ_s, μ_z). For some i, $(\lambda_s, \mu_z) \in U_i \times V_i$. However, for each $j = 1, 2, \cdots, n$, $\lambda_r \in S$. As a result, $(\lambda_s, \mu_z) \in U_i \times V_i$ as required. But $U_i \times V_i \subseteq O$ for all $i = 1 \cdots n$ and $S \times Z \subseteq \bigcup_{i=1}^{n} (U_i \times V_i)$. Therefore $S \times Z \subseteq O$. □

Theorem 4.5. *Let $(X, \tau), (Y, \sigma)$ be arbitrary L-fuzzy topological spaces and Z be a L-fuzzy locally compact Hausdorff space. Then a map $\alpha : Z \times X \to Y$ is L-fuzzy continuous if and only if $g : X \to Y^Z$ is L-fuzzy continuous where g is defined by the rule $g(\lambda_t)(z_s) = g(z_s, \lambda_t)$.*

Proof. Assume that g is L-fuzzy continuous function. Consider the function

$$Z \times X \xrightarrow{i_z \times g} Z \times Y^Z \xrightarrow{s} Y^z \times Z \xrightarrow{f} Y$$

The L-fuzzy identity function on Z is denoted by i_Z, the L-structure switching map is denoted by s, and the L-evaluation map is denoted by f. Because

$$fs(t_Z \times g)(z_s, \lambda_t) = fs(z_s, g(\lambda_t)) = f(g(\lambda_t), z_s) = g(\lambda_t)(z_s) = f(z_s, \lambda_t).$$

Implies $\alpha = fs(i_{z \times g})$

Assume, on the other hand, that α is a L-fuzzy continuous. Let λ_k represent any arbitrary fuzzy point in X. We've got $g(\lambda_k) \in Y^Z$. Consider the following subbasis element $M_{L,V} = \{h \in Y^Z : h(L) \subseteq V\}$, $L \in I^Z$ is L-fuzzy compact, and $V \in I^Y$ is L-fuzzy open containing $g(\lambda_r)$. To prove that g is a L-fuzzy continuous map, we need to discover a L-fuzzy neighborhood N of λ_k such that $\alpha(N) \subseteq M_{L,V}$; this will suffice.

We have $(g(\lambda_k))(z_v) = \alpha(z_v, \lambda_k)$ for each fuzzy point z_v in L; hence $\alpha(L \times \{\lambda_k\} \subseteq V)$, i.e., $L \times \{\lambda_v\} \subseteq \alpha^{-1}(V)$. Since $\alpha^{-1}(V)$ is a L-fuzzy open set in $Z \times X$, it is a L-fuzzy continuous. As a result, $\alpha^{-1}(V)$ is a L-fuzzy open set in $Z \times X$ that includes $L \times \{\lambda_k\}$. As a result of Theorem 4.2, a L-fuzzy neighborhood N of λ_k in X exists such that

$$L \times \{\lambda_k\} \subseteq L \times N \subseteq \alpha^{-1}(V)$$

. As a result, $\alpha(L \times N) \subseteq V$. Now,

$$\alpha(z_v, \lambda_r) = (g(\lambda_r))(z_v) \in V$$

for any $\lambda_r \in N$, and $z_v \in L$. As a result, for all $\lambda_r \in N$, $(g(\lambda_r))(L) \subseteq V$, i.e., $g(\lambda_r) \in M_{L,V}$ for all $\lambda_r \in N$. As a result, $g(N) \subseteq M_{L,V}$ as desired. □

5 Fuzzy L-exponential law

We explore some of the features of exponential law by using induced maps.

Theorem 5.1. *Let (X, τ) and (Z, σ) be L-fuzzy Hausdorff spaces that are locally compact. The function $e : Y^{Z\times X} \to (Y^Z)^X$ defined by $e(\alpha) = g$ i.e., $e(\alpha) = \hat{\alpha}(e(\alpha)(\lambda_t)(z_u) = \alpha(z_u, \lambda_t) = (\hat{\alpha}(\lambda_t))(z_u)$ for all $\alpha : Z \times X \to Y$ is a L-fuzzy homeomorphism for every L-fuzzy topological space Y.*

Proof. e is clearly L-fuzzy Surjective. Let $e(\alpha) = e(\beta)$ for e to be L-fuzzy injective. As a result, $\hat{\alpha} = \hat{\beta}$, where $\hat{\alpha}$ and $\hat{\beta}$ are the induced map of α and β, respectively. Any fuzzy point z_u in Z, as well as any fuzzy point λ_t in X. We have

$$\alpha(z_u, \lambda_t) = (\alpha^n(\lambda_t)(z_u)) = (\beta^n(\lambda_t)(z_u)) = \beta(z_u, \lambda_t)$$

thus $\alpha = \beta$.

Consider any L-fuzzy subbasis neighborhood P of α in $(Y^Z)^X$, i.e., P is of the form M_{KW}, where K is a L-fuzzy compact subset of X and$W$ is L-fuzzy open in Y^Z, to prove e is L-fuzzy continuity. We can suppose $W = M_{EV}$. Without losing generality, where E is a L-fuzzy compact subset of Z and $V \in I^Y$ is L-fuzzy open. Then $\hat{\alpha}(K) \subseteq M_{EV} = W$, as a result, we obtain $(\hat{\alpha}(K))(L) \subseteq V$ for any fuzzy point λ_t in K and all fuzzy points z_u in L, i.e., $(\hat{\alpha}(\lambda_t)(z_u)) \in V$ and so $\alpha(Z_u, \lambda_t) \in V$ L-compact in $Z \times X$ (cf. [9]) We conclude that $\alpha \in M_{E\times K,V} \subseteq Y^{Z\times X}$ because V is a L-fuzzy open set in Y.

We assert that $e(M_{E\times K}) \subseteq M_{K,W}$. Take $\gamma \in (M_{E\times K})$ to be arbitrary. Thus, $\gamma(E \times K) \subseteq V$, i.e.,$\gamma(z_u, \lambda_t) = (\hat{\gamma}(\lambda_t))(z_u)$ for all fuzzy points $z_u \in E \subseteq Z$, and $\gamma(z_u, \lambda_t) = (\hat{\gamma}(\lambda_t))(z_u)$ for all fuzzy points $\lambda_t \in K \subseteq X$. So $(\hat{\gamma}(\lambda_t)(L)) \subseteq V$ for all fuzzy points $\lambda_t \in K \subseteq X$, i.e., $(\hat{\gamma}(\lambda_t) \in M_{E,V} = W$ for all fuzzy points $\lambda_t \in K \subseteq X$, i.e., $\hat{\gamma}(\lambda_t) \in M_{E,V} = W$ for all fuzzy points $\lambda_t \in K \subseteq X$. As a result, we have $\hat{\gamma}(K) \subseteq W$, i.e., $\hat{\gamma} = e(\gamma) \in M_{KW}$ for any $\gamma \in M_{E\times K}, V$. As a result, $e(M_{E\times K}) \subseteq M_{K,W}$. This shows that e is a L-fuzzy continuous.

The following L-fuzzy evaluation maps are used to show the L-fuzzy continuity of e^{-1}. $e' : (Y^Z)^X \times X \to Y^Z$ is defined by $e'(\hat{\alpha}, \lambda_t) = \hat{\alpha}(\lambda_t)$, where $\hat{\alpha} \in (Y^Z)^X$ and λ_t are any fuzzy points in X and $e'' : (Y^Z) \times Z \to Y$ is defined by $e''(\hat{\beta}, z_u) = \hat{\beta}(z_u)$, where $\beta \in Y^Z$ and $z_u \in Z$. Let ϕ denote the composition of the following $\Im^*$-structure continuous function.

$$(Z \times X) \times (Y^Z)^X \xrightarrow{F} (Y^Z)^X \times (Z \times X) \xrightarrow{i\times\tilde{f}} (Y^Z)^X \times (X \times Z) =$$

$$((Y^X) \times X) \times Z \xrightarrow{e'\times i_z} Y^Z \times Z \xrightarrow{e''} Y$$

The L-fuzzy identity maps on $(Y^Z)^X$ and Z are denoted by i and i_z, while the L-fuzzy switching maps are denoted by F and $\tilde{f}$. As a result, $\phi : (Z \times X) \times (Y^Z)^X \to Y$, i.e., $\phi \in Y^{(Z\times X)\times(Y^Z)^X}$.

We look at the L-fuzzy map

$$\tilde{e} : Y^{(Z\times X)\times(Y^Z)^X} \to Y^{(Z\times X)(Y^Z)^X},$$

i.e., $\tilde{e}(\phi) \in Y^{(Z\times X)(Y^Z)^X}$. We've got a L-fuzzy continuous map $\tilde{e}(\phi) : (Y^Z)^X \to Y^{Z\times X}$
Now, checking that $(\tilde{e}(\phi) \circ e)(\alpha)(z_u, \lambda_t) = \alpha(z_u, \lambda_t)$, therefore $\tilde{e}(\phi) \circ e = L$-structure identity. It is also a routine matter to check that $(e \circ \tilde{e}(\phi))(\hat{\beta})(\lambda_t)(z_u) = \hat{\beta}(\lambda_t)(z_u)$; therefore $e \circ \tilde{e}(\phi) =$ L-fuzzy identity for any $\hat{\beta} \in (Y^Z)^X$ and fuzzy points $z_u \in Z$, $\lambda_t \in X$. The demonstration that e is a L-fuzzy homeomorphism is now complete.

Definition 5.2. The L-fuzzy map e in Theorem 5.1 is called L-fuzzy exponential law.

Corollary 5.3. Let X, Y, Z be locally L-fuzzy compact Hausdorff spaces. Then the map $u : Y^X \times Z^Y \to Z^X$ defined by $u(f, g) = g \circ f$ is L-fuzzy continuous.

Proof. Consider the following compositions:

$$X \times Y^x \times Z^Y \xrightarrow{T} Y^X \times Z^Y \times X \xrightarrow{t\times i_u} Z^Y \times Y^X \times X \to Z^Y \times (Y^X \times X) \xrightarrow{i\times e_1} Z^Y \times Y \xrightarrow{v} Z$$

where T, t denote the L-fuzzy switching maps, i_x and i denote the L-fuzzy identity functions on X and Z^Y, respectively, and e_1, e_2 denote the L-fuzzy evaluation maps. Let $\varphi = e_2 \circ (i \times e_1) \circ (t \times i_X) \circ T$. By Theorem 5.1 we have a L-fuzzy exponential map $e : Z^X \times Y^X \times Z^Y \to (Z^X)^{Y^X \times Z^Y}$.

Let $\varphi \in Z^{X\times Y^X \times Z^Y}$, $e(\varphi) \in (Z^X)^{Y^X \times Z^Y}$. Again we consider let $u = e(\varphi)$ is L-fuzzy continuous. For $f \in Y^X, g \in Z^Y$ and for any L-fuzzy point x_t in X, it is easy to see that $u(fg)(x_t) = g(f(x_t))$. □

6 Conclusion

In this work, the concept of L-fuzzy open compact topology and L-fuzzy exponential law have been introduced along with some basic theories. This work will lay the foundation for further research on L-fuzzy open compact topology. We hope to build a concept of L-fuzzy higher homotopy groups and fuzzy universal covering space using this concept. Further, this topic can be expanded in fuzzy category theory in future. □

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